and if it rebounds vertically upward, then

$$\cot \psi = (2n + 1) \tan \varphi.$$

### SOLUTION BY IMMANUEL KLAUFF, Chicago, Ill.

The equation of the path is, if we let $\varphi + \psi = \alpha$,

$$y = x \tan \alpha - \frac{gx^2}{2v_0^2 \cos^2 \alpha}.$$

The given inclined plane intersects this parabola at $P_1$ and the slope of the parabola at $P_1$ is

$$\left(\frac{dy}{dx}\right)_{P_1} = 2 \tan \varphi - \tan \alpha = \tan \omega.$$

The angle of inclination of the first rebound is

$$\omega_1 + \varphi = \gamma_1.$$

where $\omega_1$ is the complement of $\omega$. We have

$$\tan \gamma_1 = \frac{\tan \omega_1 + \tan \varphi}{1 - \tan \omega_1 \tan \varphi} = \frac{\tan \varphi - \tan \omega}{1 + \tan \omega \tan \varphi} = \frac{\tan \alpha - \tan \varphi}{1 - 2 \tan^2 \varphi + \tan \varphi \tan \varphi} = \frac{\tan \psi}{1 - 2 \tan \psi \tan \varphi}.$$

The analysis for the subsequent rebounds is the same; the angle of projection with respect to the inclined plane is for the $n$th rebound

$$\gamma_n = \tan^{-1} \left[ \frac{\dfrac{\tan \psi}{1 - 2(n-1) \tan \psi \tan \varphi}}{1 - \dfrac{2 \tan \psi \tan \varphi}{1 - 2(n-1) \tan \psi \tan \varphi}} \right] = \tan^{-1} \left( \frac{\tan \psi}{1 - 2n \tan \psi \tan \varphi} \right).$$

If the ball rebounds vertically, we have

$$\tan (\gamma_n + \varphi) = \infty, \qquad \text{or} \qquad 1 - \frac{\tan \psi \tan \varphi}{1 - 2n \tan \psi \tan \varphi} = 0.$$

Whence, $1 - (2n + 1) \tan \psi \tan \varphi = 0$; and $\cot \psi = (2n + 1) \tan \varphi$.

Also solved by H. S. UHLER, HORACE OLSON, and the PROPOSER.

## NUMBER THEORY.

**185. Proposed by R. D. CARMICHAEL, Bloomington, Indiana.**

Obtain the complete solution of the equation $\phi(p^\alpha) = \phi(q^\beta)$ where $\phi$ denotes Euler's $\phi$-function, $p$ and $q$ are unknown primes, and $\alpha$ and $\beta$ are unknown integers.

### CRITICISM BY T. H. GRONWALL, New York City.

The solution of this problem on page 227, Volume XX, September, 1913, is open to criticism. From the equation:

$$p^{\alpha-1}(p - 1) = q^{\beta-1}(q - 1),$$

where $p$ and $q$ are primes, and $p > q$, it follows that

$$p - 1 = \lambda q^{\beta-1}, \qquad q - 1 = \lambda p^{\alpha-1}$$

where $\lambda$ is a positive integer. Since $q < p$, the second equation shows that $\alpha = 1$, and consequently $\lambda = q - 1$,

$$p = 1 + (q - 1)q^{\beta-1},$$

and here the prime $q$ and the exponent $\beta$ have to be chosen so that $p$ becomes a prime. The error consists in assuming $\lambda = 1$ and hence $q = 2$; that this does not give all the solutions, is evident from the examples:

$$q = 2,\ \beta = 2,\ 3,\ 5 \qquad q = 3,\ \beta = 2,\ 3,\ 5 \qquad q = 5,\ \beta = 3,\ 5$$
$$p = 3,\ 5,\ 17 \qquad p = 7,\ 19,\ 163 \qquad p = 101,\ 2501.$$

**214.** (April, 1914.) **Proposed by A. J. KEMPNER, University of Illinois.**

Let $a$ be a positive integer $\geqq 2$, and let $T(n)$ denote the number of distinct divisors of the positive integer $n$, including both 1 and $n$, so that $T(1) = 1$, $T(2) = 2$, $T(3) = 2$, $T(4) = 3$, $\cdots$. Show that

$$\sum_{n=1}^{n=\infty} T(n)/a^n = \sum_{n=1}^{n=\infty} 1/(a^n - 1).$$

The special case $a = 10$ gives, as is easily seen:

$$9 \sum_{n=1}^{n=\infty} \frac{T(n)}{10^n} = \frac{1}{1} + \frac{1}{11} + \frac{1}{111} + \frac{1}{1111} + \cdots.$$

SOLUTION BY FRANK IRWIN, University of California.

We prove the proposition first for the special case, $a = 10$. We have,

$$1/9 = .11111111\cdots, \qquad 1/99 = .01010101\cdots,$$
$$1/999 = .001001001\cdots, \qquad 1/9999 = .00010001\cdots,$$

and so on.

Let us determine the sum of all the digits in the $n$th decimal place. Since the $k$th row of the above array reads

$$1/99\cdots 99 = .00\cdots 00100\cdots 001\cdots,$$

with $k$ 9's on the left and $(k - 1)$ 0's in each recurring period on the right, it follows that we get a 1 in the $n$th decimal place whenever $k$ is a divisor of $n$, and otherwise a zero. We have, then, $T(n)$ units in this place, and since a unit there has the value $1/10^n$, their sum is $T(n)/10^n$; and the sum of our series, $1/9 + 1/99 + 1/999 + \cdots$, is equal to

$$\sum_{n=1}^{n=\infty} T(n)/10^n,$$

as was to be proved.

(It is clear that, regarding the array as a double series, we have a right to put the sum by rows equal to the sum by columns.)

In the general case, where we have any $a$, we need merely suppose the decimals above written in the scale of $a$, instead of in that of 10,

$$1/(a - 1) = .1111\cdots, \quad 1/(a^2 - 1) = .0101\cdots, \text{ etc.},$$

and a like argument holds.

# QUESTIONS AND DISCUSSIONS.

**[Send all Communications to U. G. MITCHELL, University of Kansas, Lawrence, Kans.]**

## DISCUSSIONS.

### I. RELATING TO NAPIER'S LOGARITHMIC CONCEPT.

BY H. S. CARSLAW, University of Sydney, Australia.

In the March number of the MONTHLY, page 71, Professor Cajori takes exception to the following remark, contained in a paper of mine in the *Mathematical Gazette* (Vol. VIII, page 77):